Hurricane Eyewall Slope as Determined from Airborne Radar Reflectivity Data: Composites and Case Studies

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ABSTRACT

Understanding and predicting the evolution of the tropical cyclone (TC) inner core continues to be a major research focus in tropical meteorology. Eyewall slope and its relationship to intensity and intensity change is one example that has been insufficiently studied. Accordingly, in this study, radar reflectivity data are used to quantify and analyze the azimuthal average and variance of eyewall slopes from 124 flight legs among 15 Atlantic TCs from 2004 to 2011. The slopes from each flight leg are averaged into 6-h increments around the best-track times to allow for a comparison of slope and best-track intensity. A statistically significant relationship is found between both the azimuthal mean slope and pressure and between slope and wind. In addition, several individual TCs show higher correlation between slope and intensity, and TCS with both relatively high and low correlations are examined in case studies. In addition, a correlation is found between slope and radar-based eye size at 2 km, but size shows little correlation with intensity. There is also a tendency for the eyewall to tilt downshear by an average of approximately 10°. In addition, the upper eyewall slopes more sharply than the lower eyewall in about three-quarters of the cases. Analysis of case studies discusses the potential effects on eyewall slope of both inner-core and environmental processes, such as vertical shear, ocean heat content, and eyewall replacement cycles. These results indicate that eyewall slope is an important measure of TC inner-core structure, and may prove useful for future study of the processes that drive changes in the TC core.

1. Introduction

The eye of a tropical cyclone (TC) is one of the well-known aspects of TC structure, and was observed (e.g., Ballou 1892) long before aircraft reconnaissance or satellite allowed for more detailed study. Despite the long observational record of the eye, many aspects of core structure are just now becoming better understood. Willoughby (1979, 1988) and Shapiro and Willoughby (1982) presented a theoretical model of the TC secondary circulation (within an axisymmetric, balanced framework), with radial inflow at low levels, ascent and parameterized latent heat release in the updrafts in the eyewall, forced descent within the eye (if the eyewall is tilted), and outflow at upper levels. In addition, this model provides a theoretical basis for the observed contraction of the eyewall as a hurricane intensifies. It also describes secondary eyewall formation and replacement (Willoughby et al. 1982), first documented observationally by Fortner (1958). Research is now beginning to further illuminate the processes behind this phenomenon, and Kossin and Sitkowski (2009) provided the first tool for objectively forecasting the development of secondary eyewalls. Piech and Hart (2008) compiled a climatology of reconnaissance vortex messages from the Atlantic basin between 1989 and 2006 (later updated by Cossuth and Hart 2012), and used this dataset to analyze the relationship between eye thermodynamics and eye structure. Vigh et al. (2012) provided the first climatology documenting the formation of TC eyes and the conditions associated with eye formation.

Although the TC inner-core region consisting of the eye and eyewall is frequently observed, many of the numerous small-scale processes in the core are poorly understood because of the unusually complex physical interactions involved. One such phenomenon is the slope of the eyewall, which was first documented observationally in the 1950s. Malkus (1958) theorized that the outward slope of the eyewall is caused by outflow,
causing the tops of the cumulonimbus clouds to spread outward. That study also noted that the cumulonimbus clouds that form the eyewall grow vertically at first and then start to slant as the updraft decays. Malkus and Riehl (1960) assumed a vertical eyewall, but hypothesized that a more sloped eyewall would allow for the extreme pressure falls that are seen in some hurricanes, as this structure would lead to an enhanced warm anomaly in the core. Riehl and Malkus (1961) also hypothesized that an eyewall that sloped outward over the rain core of the eyewall would reduce the TC's central pressure. As part of the aforementioned Shapiro and Willoughby (1982) model of the secondary circulation in a TC, the Sawyer–Eliassen axisymmetric balance model for TCs provided the first theoretical basis for why the eyewall must slope. This theory shows that the secondary circulation induced by a heat source in a hurricane in gradient wind balance has an updraft that flows along angular momentum surfaces. To satisfy thermal wind balance in a warm-core vortex, which weakens with height, these momentum surfaces must slope outward with height. However, this model (as its name implies) only applies to an idealized axisymmetric framework and, therefore, does not directly explain eyewall slope in the full 3D structure of a TC, nor does it account for asymmetries in slope resulting from factors such as vertical shear.

More recent observational studies discussing eyewall slope

Shea and Gray (1973) first observationally analyzed the slope of the eyewall for multiple storms. That study focused on the slope of the radius of maximum wind (RMW) and analyzed aircraft observations for 21 hurricanes from 1957 to 1969. They found that the RMW tended to slope outward more in the weaker storms and was more upright in the stronger storms. They argued that the vertical slope of the RMW is caused by convection in the inner-core region, and that stronger convection led to the more upright RMW. A limitation of that study was coarse vertical resolution, as some of the slope measurements were made from flights at only two levels and, therefore, did not show all of the details of the changes in slope with height (Stern and Nolan 2009, hereafter SN09; Stern and Nolan 2011).

Jorgensen (1984) used radar reflectivity data to analyze the slope, but was limited to 10 flights into four different hurricanes. For the limited cases available, he found that the 10-dBZ reflectivity contour sloped outward with height, and that the eyewall was more upright for stronger storms and smaller eyes. For Hurricane Allen (1980), as the eye diameter halved, Jorgensen (1984) found that the eyewall slope decreased from 60° (from vertical) to 45°. This was also illustrated in Marks (1985), which studied the structure of precipitation in the same TC. The other interesting observation from Jorgensen (1984) was that the maximum radar reflectivity was displaced from the RMW, suggesting that it is possible for the slope of the edge of the eyewall to be different from the slope of the RMW.

Corbosiero et al. (2005) also looked at the slope of the 10-dBZ contour for Hurricane Elena (1985), as part of a broader study of that TC’s structure. That study noted the differences in slope between the lower part of the storm (0–6 km) and the upper part (6–12 km), with the eyewall being less upright at upper levels. Aberson et al. (2006) noted another slope asymmetry in Hurricane Erin (2001): a difference in slope between two sides of the eyewall that was attributed to vertical shear. Schubert and McNoldy (2010) noted an eyewall updraft slope of approximately 45° in Typhoon Choi-Wan during 2009. Through a solution of the transverse circulation in the previously mentioned balanced vortex model (e.g., Shapiro and Willoughby 1982), they showed how the baroclinicity of the TC vortex is important in regulating the slope (although not the strength) of the eyewall updraft.

SN09 provided the most comprehensive analysis of eyewall slope to date. That study also used radar data to observe eyewall slope, focusing on the RMW vertical structure and slope as determined from Doppler radar velocity data from seven hurricanes from 2004 to 2006. In contrast to some of the previous studies (particularly Shea and Gray 1973), SN09 showed little relationship between the slope of the RMW and TC intensity, although they did find a relationship between the size of the RMW at 2 km and the slope of the RMW. The lack of a slope–intensity relationship was shown for individual storms as well as over all cases. Interestingly, SN09 also found that the RMW did not exactly follow an angular momentum surface in several cases, although they showed that the approximation is very close to theory within the framework of maximum potential intensity (MPI; Emanuel 1986). Rogers et al. (2012) also briefly analyzed RMW slope as part of a broader study of TC structure using airborne radar data, with results similar to those of SN09. Table 1 summarizes the slope definitions and number of storms analyzed used in these previous studies as well as in the current study (discussed below).

Although SN09 found little relationship between RMW slope and intensity, the edge of the eyewall may provide a measure of the slope that is dynamically different and, therefore, have a different relationship with storm structure and intensity. In addition, previous studies have focused either on azimuthal means or
samples from one part of a TC. Thus, although some have discussed azimuthal or vertical variation in slope within a given TC, there has been insufficient work to comprehensively quantify these asymmetries for multiple cases. Accordingly, we build upon the foundation of work previously discussed and use radar reflectivity data to determine the slope of the eyewall edge for 15 hurricanes. The methodology used (section 2) quantifies observed asymmetries in slope and provides a large enough dataset for comprehensive analysis of these asymmetries (section 3). We also analyze the relationship between the slope of the edge of the eyewall and the intensity of the TC. The analysis performed here considers composites from multiple storms to improve statistical robustness and, then, looks at a detailed analysis of case studies (section 4). In those case studies we analyze how the slope may relate to different physical processes within the TC core and its interaction with the environment. Thus, this dataset expands on previous analyses of eyewall slope not only by allowing us to consider new aspects of slope and its relationship to storm structure, but also by giving us the opportunity to contrast and compare composite results to specific storms.

2. Data and methodology

a. Algorithm for slope calculation

In this study, the focus is on the slope of the edge of the eyewall and its relationship with the inner-core structure and intensity of the storm. Although it may not be possible to exactly pinpoint the edge of the eyewall updraft, radar reflectivity can be used to estimate its location. Our approach is similar to the methodology employed in Jorgensen (1984) and Corbosiero et al. (2005). Starting from the center of the storm, the algorithm searches radially outward in each azimuthal direction at each vertical level until it reaches a threshold value of reflectivity: 20 dBZ for National Oceanic and Atmospheric Administration (NOAA) P3 flights by the Hurricane Research Division (HRD; described below), and 10 dBZ for data from two field campaigns (also described below). The threshold used is different between the datasets because of differences in the radar frequency, as the Airborne Second Generation Precipitation Radar (APR-2) especially appeared to suffer some attenuation at low levels (S. Durden 2012, personal communication). The threshold was intended to be an approximation of the interface between the eye’s clear air and the convection associated with the eyewall updraft. With the updraft approximately located within the region of maximum reflectivity (generally greater than 30 dBZ), and the eye having reflectivity less than 5 dBZ in most cases, the eye–eyewall interface can be reasonably approximated in between these extremes. Subjective analysis of several cases showed that the above thresholds were reasonable for approximating the eye–eyewall interface.

The algorithm above was only applied between heights of 2 and 11 km. The lower boundary is intended to prevent contamination from boundary layer processes such as the radial inflow into the eye, and is the same threshold used by SN09. The upper-level boundary was chosen because of a dramatically decreasing sample size for elevations above 11 km. If the 20-dBZ contour does not extend up to 11 km, the algorithm stops at the level at which the threshold is last defined. For cases where a secondary eyewall is observed, the algorithm only considers the inner eyewall, and stops at the top of this feature. After performing these calculations, the algorithm outputs a set of points identifying the horizontal and vertical dimensions of the edge of the eyewall, and

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slope definition</th>
<th>No. of storms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shea and Gray (1973)</td>
<td>RMW</td>
<td>21</td>
</tr>
<tr>
<td>Jorgensen (1984)</td>
<td>10-dBZ contour</td>
<td>1</td>
</tr>
<tr>
<td>Corbosiero et al. (2005)</td>
<td>10-dBZ contour</td>
<td>1</td>
</tr>
<tr>
<td>Stern and Nolan (2009)</td>
<td>RMW</td>
<td>7</td>
</tr>
<tr>
<td>Schubert and McNoldy (2010)</td>
<td>Eyewall updraft</td>
<td>1</td>
</tr>
<tr>
<td>Rogers et al. (2012)</td>
<td>RMW</td>
<td>8</td>
</tr>
<tr>
<td>Current study</td>
<td>20-dBZ contour</td>
<td>15</td>
</tr>
</tbody>
</table>
uses least squares linear regression to determine the slope of the "best fit" line passing through those points. The slope is expressed as the angle from upright (in degrees), where a slope of 0° would indicate a perfectly upright eyewall. It should be noted that this definition of slope differs from that of SN09, which defined slope as the ratio $\frac{\delta r}{\delta z}$. We found that the choice of slope definition did not alter the results in a statistically significant manner (not shown) and demonstrates robustness of the relationships found in both studies. An example of the slope-calculation process is given in Fig. 1.

### Radar datasets used

The radar data for this study come from several different sources. Radar data for Hurricane Earl (2010) were provided from the National Aeronautics and Space Administration (NASA) Genesis and Rapid Intensification Processes (GRIP) experiment (Zipser et al. 2012). The data from this campaign were collected using the APR-2, a dual-frequency (13 and 35 GHz) Doppler radar system (Sadowy et al. 2003; Haddad et al. 2006). Radar data for Hurricane Erin (2001) came from

![Figure 2](image-url)
NASA’s Fourth Convection and Moisture Experiment (CAMEX-4; Kakar et al. 2006). The radar platform used in CAMEX-4 was the ER-2 Doppler radar (EDOP), which is a 9.6-GHz radar in the nose of the ER-2 aircraft (Caylor et al. 1994; Halverson et al. 2006). The remainder of cases came from research flights by NOAA/HRD using the tail radar of their P3 aircraft [a 9.3-GHz Doppler radar; see Gamache (1997) and Lee and Marks (2000)]. These data have been processed by HRD into three-dimensional grids (available online at http://www.aoml.noaa.gov/hrd/data_sub/radar.html) with 0.5-km vertical resolution. The horizontal resolution of the data ranges from 2 to 4.5 km for different storms. While this vertical resolution. The horizontal resolution of the data ranges from 2 to 4.5 km for different storms. While this may lead to slight differences in the slope calculations, the differences are likely insignificant. All data were converted into polar coordinates, and slopes were calculated every 5° azimuthally, starting at 0° (east of the center) and rotating counterclockwise to 355°. The slopes were then averaged to give the azimuthal mean slope for that flight leg. The azimuthal coverage was nearly complete in all of the cases analyzed, except outside of the inner-core region, which is irrelevant for this study. For the GRIP and CAMEX field experiments, the geometry of the APR-2 and EDOP radar scans prevented the computation of azimuthal means; thus, for consistency the results for Hurricanes Earl (2010) and Erin (2001) are not included in the composite results, because the slopes were calculated from only one cross section rather than taking azimuthal means as in the other cases.

### Table 3. Summary of storms used for slope calculations.

<table>
<thead>
<tr>
<th>Storm</th>
<th>Year</th>
<th>Radar source</th>
<th>No. of cases (best track)</th>
<th>Mean slope (± SE)</th>
<th>Letter in scatterplots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erin</td>
<td>2001</td>
<td>EDOP</td>
<td>2 (1)</td>
<td>76.7° (—)</td>
<td>—</td>
</tr>
<tr>
<td>Frances</td>
<td>2004</td>
<td>NOAA P3</td>
<td>9 (4)</td>
<td>58.2° (± 6.9°)</td>
<td>A</td>
</tr>
<tr>
<td>Ivan</td>
<td>2004</td>
<td>NOAA P3</td>
<td>24 (9)</td>
<td>53.8° (± 4.3°)</td>
<td>B</td>
</tr>
<tr>
<td>Jeanne</td>
<td>2004</td>
<td>NOAA P3</td>
<td>11 (4)</td>
<td>64.2° (± 4.6°)</td>
<td>C</td>
</tr>
<tr>
<td>Dennis</td>
<td>2005</td>
<td>NOAA P3</td>
<td>3 (3)</td>
<td>47.9° (± 6.6°)</td>
<td>D</td>
</tr>
<tr>
<td>Katrina</td>
<td>2005</td>
<td>NOAA P3</td>
<td>4 (3)</td>
<td>55.2° (± 2.8°)</td>
<td>E</td>
</tr>
<tr>
<td>Ophelia</td>
<td>2005</td>
<td>NOAA P3</td>
<td>2 (2)</td>
<td>56.3° (± 9.0°)</td>
<td>F</td>
</tr>
<tr>
<td>Rita</td>
<td>2005</td>
<td>NOAA P3</td>
<td>5 (3)</td>
<td>50.3° (± 3.2°)</td>
<td>G</td>
</tr>
<tr>
<td>Wilma</td>
<td>2005</td>
<td>NOAA P3</td>
<td>1 (1)</td>
<td>66.5° (—)</td>
<td>H</td>
</tr>
<tr>
<td>Helene</td>
<td>2006</td>
<td>NOAA P3</td>
<td>9 (3)</td>
<td>64.5° (± 4.8°)</td>
<td>I</td>
</tr>
<tr>
<td>Felix</td>
<td>2007</td>
<td>NOAA P3</td>
<td>11 (4)</td>
<td>48.0° (± 8.2°)</td>
<td>J</td>
</tr>
<tr>
<td>Dolly</td>
<td>2008</td>
<td>NOAA P3</td>
<td>2 (1)</td>
<td>70.1° (—)</td>
<td>K</td>
</tr>
<tr>
<td>Gustav</td>
<td>2008</td>
<td>NOAA P3</td>
<td>15 (7)</td>
<td>67.2° (± 1.0°)</td>
<td>L</td>
</tr>
<tr>
<td>Ike</td>
<td>2008</td>
<td>NOAA P3</td>
<td>10 (4)</td>
<td>57.7° (± 4.8°)</td>
<td>M</td>
</tr>
<tr>
<td>Paloma</td>
<td>2008</td>
<td>NOAA P3</td>
<td>12 (3)</td>
<td>61.2° (± 4.9°)</td>
<td>N</td>
</tr>
<tr>
<td>Earl</td>
<td>2010</td>
<td>NOAA P3</td>
<td>3 (3)</td>
<td>58.2° (± 6.9°)</td>
<td>O</td>
</tr>
<tr>
<td>Mean</td>
<td>—</td>
<td>—</td>
<td>8 (4)</td>
<td>58.5° (± 5.5°)</td>
<td>—</td>
</tr>
</tbody>
</table>

The composite data consist of 124 different flight legs into 15 TCs from 2001 to 2011. Figure 2a is a plot of the locations of all of the slope measurements, where the letters indicate different storms (further detailed in Table 3). The size of the letter is proportional to the eyewall slope, with smaller letters corresponding to more upright eyewalls. We calculated slopes for all 124 cases available, but also wanted a measure of slope that was less prone to short-term fluctuation and more indicative of overall storm structure. This was done by averaging the slope measurements in 6-h windows around the nearest “best track” (BT) time of 0000, 0600, 1200, or 1800 UTC. For example, slope measurements at 1000, 1100, and 1400 UTC would be averaged to give a 1200 UTC slope. While 6-h spacing does not assure statistical independence, this 6-h time scale allows a more straightforward comparison to BT intensity.

Figure 2b is identical to Fig. 2a, except that it shows the slopes averaged around the BT times. Table 3 is a summary of all of the storms analyzed, including the year, radar platform used, number of measurements, average slope for each storm, and standard error of the mean slope. The intensities over which the eyewall slope was measured ranged from a minimum of 55 kt (1 kt ~ 0.5 m s⁻¹) in two cases (Dolly and Ophelia) to a maximum of 150 kt (Felix). The 55-kt lower bound is consistent with Vigh et al. (2012), who found that many eyes form when a storm is just below hurricane strength. Thus, our dataset includes both newly formed and developed eyewalls, although, as in SN09, the dataset is biased toward storms with higher intensities, which tend to have more coverage from aircraft.

### 3. Composite results

After compiling the dataset of eyewall slopes, we next examine the characteristics of the composite dataset. The overall distribution of azimuthal-mean slopes is shown in section 3a, the relationship of slope to intensity in sections 3b and 3c, and to eye size in section 3d. The section concludes with analyses and discussions of azimuthal asymmetry of slope in section 3e and vertical change in slope in section 3f.

#### a. Distribution of slopes

Figure 3a shows the edge of the eyewall as defined by the 20-dBZ contour from 2 to 11 km for all 124 flight legs analyzed. Figure 3b shows the mean profile and interquartile range. Note the wide range of sizes, heights, and slopes. The colors in Fig. 3a show storm intensity, with
warmer colors indicating stronger storms. There seems to be a tendency for stronger storms to have smaller eyes and more upright eyewalls, although there is considerable spread. This relationship is discussed in more detail in section 3b. To show the profiles without the influence of eye size, the eyewalls were next plotted relative to the size of the eye at 2-km height (also defined by the 20-dBZ contour) in Fig. 3c. Figure 3d shows the mean “relative” eyewall and interquartile range. Removing the eye size better highlights the tendency for outward slope, especially at upper levels. Once again, the coloring by intensity does seem to indicate some tendency for stronger storms to have more upright eyewalls. This relationship is explored in much more detail in later sections.

A histogram showing the distribution of slopes for the cases examined is shown in Fig. 4. The bars show the distribution for all 124 cases analyzed, and the line shows the distribution after averaging around the BT times. In both cases, the peak of the distribution is around 70° from vertical, although the BT distribution has smaller tails due to the temporal binning. Some of the factors influencing slope changes in individual storms are discussed later. Although the peak of the distribution is around 70°, the mean is approximately 58°, showing the skew toward more upright eyewalls. It should be noted that these slopes tend to be greater than the RMW slopes calculated by SN09 (possibly resulting from differences in the vertical limit used), although the range is approximately the same.

b. Mean relationship between slope and intensity using best-track results

Prior research has disagreed on the existence of a relationship between RMW slope and intensity; Shea and...
Gray (1973) suggested a connection but the more comprehensive work of SN09 suggested little relationship. Utilizing the database collected here, which notably uses a different definition for eyewall slope than SN09, we reexamine the disputed relationship. Both maximum wind and minimum pressure were used for the intensity comparison. Wind is a more theoretically based representation of the intensity of the storm (Emanuel 1986), but is difficult to directly measure. While pressure may not be a direct measurement of storm intensity, it is physically related to the strength of the TC warm core and is also more directly measured when aircraft are present. Thus, pressure tends to be a more reliable measurement than wind, the latter of which tends to be underestimated by aircraft observations (Uhlhorn and Nolan 2012). Given that the measurements of pressure, wind, and the associated radar scans are often separated by up to several hours, the slope measurements were averaged around the BT times and compared to the BT intensity. As discussed earlier, this averaging technique reduced some of the potential noise caused by short-term fluctuations in the slope and core structure, and allowed comparison of eyewall slope to wind, since the BT incorporates all available data (including data not available in real time).

Figure 5 shows the statistically significant relationship (correlation coefficient $r = 0.33$; significance level $p < 0.01$) between eyewall slope and BT minimum central pressure $P_{\text{min}}$ for all of the cases analyzed. Here, $r$ is the linear correlation coefficient, and the $p$ value comes from a $t$ test. The letters indicate different storms (Table 3), and the distribution of the letters illustrates how the relationship is not dominated by storms with (several) relatively constant slope–intensity combinations. The overall relationship between eyewall slope and storm intensity shown in Fig. 5 is most notably violated by storms where the pressure was low but the eyewall was less upright. Such a pattern may indicate a phase difference between changes in inner-core structure and the pressure response in the TC, or it may simply indicate a less common structural existence. It is not clear from this dataset whether changes in slope are typically a cause of, or result of, other changes in inner-core structure. In section 4 we discuss this relationship for several cases. For the composite data shown here, the lag correlation between pressure and slope was relatively constant between 6 h ($r = 0.33$) and 12 h ($r = 0.31$), and began to degrade at 18 h ($r = 0.27$) and 24 h ($r = 0.25$).

The slope is also correlated with the maximum sustained BT wind (Fig. 6). In fact, the slope–wind relationship ($r = -0.41; p < 0.01$) is stronger than the slope–pressure relationship ($r = 0.33; p = 0.014$). However, this difference may not be statistically significant. The lag correlations between slope and wind show a similar pattern to those between slope and pressure, with correlation staying relatively constant out to 12 h before dropping off around 18 h. Figure 7 summarizes the correlations between slope and intensity for the 24 h preceding and after each slope measurement (for wind). The relatively slow and symmetric dropoff of correlation on either side of time zero in Fig. 7 suggests that the overall relationship is unlikely to have predictive value. Yet, when examined on a case-by-case basis, there is
evidence that eyewall slope is connected to inner-core processes responsible for intensity change, as discussed in section 4.

c. Relationships between slope and intensity within the life cycles of individual storms

While the correlation between slope and intensity is statistically significant, there is a lot of variance that should be explained by quantifying the relationship within each storm. Table 4 shows the correlations (and significance) between slope and intensity (maximum wind speed) for all of the storms analyzed that had more than two BT points. There is a wide range of correlations, but with most of the storms only having three or four BT cases, expectedly only two are significant at the 90% level and none is significant at the 95% level. Felix (2007), with the lowest p value (0.07), is analyzed as a case study (discussed later). Smaller-scale inner-core changes (such as eyewall replacement cycles) may be responsible for reducing the strength of the intensity–slope relationship, as discussed later in the case of Ivan (2004). It would be expected that more of these short-term inner-core changes would progressively reduce the correlation, which is likely why Hurricanes Ivan and Gustav (which had the most BT cases and were spread out over several days) both showed low correlation.

d. Relationship between eyewall slope and eye size

Some prior studies (e.g., Shea and Gray 1973; Jorgensen 1984; Marks 1985) suggested that storms with smaller eyes tended to have more upright eyewalls. SN09 also found an apparent relationship between the slope of the RMW and the size of the RMW at 2-km height. We next investigate the relationship between the azimuthal mean slope and the size of the eye at 2 km (both defined by the 20-dBZ threshold). Figure 8 is a scatterplot of eyewall slope versus eye size and shows a significant correlation ($r = 0.56$; $p < 0.01$). The mean correlation has notable outliers, in particular combinations of small eyes and large slope. Closer examination of these outliers shows that many of them were from Hurricanes Ike (2008) and Irene (2011), and happened to coincide with different periods of secondary eyewall formation in both storms, as determined by examining radar and microwave data (not shown). In these transition periods in the storm core, the eye was small, but the inner eyewall was unusually sloped (usually $60^\circ$-$70^\circ$), possibly as a result of the growth of the outer eyewall and the corresponding weakening of the upward inertia of parcels in the inner eyewall updraft.

<table>
<thead>
<tr>
<th>Storm (year)</th>
<th>No. of BT cases</th>
<th>Slope–wind $r$</th>
<th>$p$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frances (2004)</td>
<td>4</td>
<td>-0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>Ivan (2004)</td>
<td>9</td>
<td>0.19</td>
<td>0.63</td>
</tr>
<tr>
<td>Jeanne (2004)</td>
<td>4</td>
<td>-0.81</td>
<td>0.20</td>
</tr>
<tr>
<td>Dennis (2005)</td>
<td>3</td>
<td>-0.79</td>
<td>0.42</td>
</tr>
<tr>
<td>Katrina (2005)</td>
<td>3</td>
<td>0.98</td>
<td>0.13</td>
</tr>
<tr>
<td>Rita (2005)</td>
<td>3</td>
<td>0.99</td>
<td>0.10</td>
</tr>
<tr>
<td>Helene (2006)</td>
<td>3</td>
<td>0.48</td>
<td>0.68</td>
</tr>
<tr>
<td>Felix (2007)</td>
<td>4</td>
<td>-0.93</td>
<td>0.07</td>
</tr>
<tr>
<td>Gustav (2008)</td>
<td>8</td>
<td>-0.22</td>
<td>0.64</td>
</tr>
<tr>
<td>Ike (2008)</td>
<td>4</td>
<td>-0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>Paloma (2008)</td>
<td>3</td>
<td>-0.89</td>
<td>0.26</td>
</tr>
<tr>
<td>Irene (2011)</td>
<td>3</td>
<td>0.63</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Finally, it is also worth noting that our analysis showed little relationship between the size of the eye at 2 km and the intensity of the storm ($|r| \approx 0.11$ between eye size and pressure or wind). This result is also consistent with the results of SN09 for the size of the RMW. It is also consistent to some extent with the work of Kimball and Mulekar (2004), which found similar distributions of eye size for category 3–4 storms, which make up 40% of the cases here. As noted in SN09, a relationship between eye size and intensity would only be expected within a given storm, and this relationship is also sometimes muddled by eyewall replacement processes. Because the overall correlation between size and intensity is low, it is clear that the eye size–slope relationship is insufficient to explain the relationship between slope and intensity.

e. Asymmetric slopes and relationship to shear

Significant azimuthal variance in the slope of the eyewall was often noted in the cases analyzed. These differences in eyewall slope across the eye have been noted in other studies and attributed to environmental shear (e.g., Malkus 1958; Halverson et al. 2006). The current study investigated the relationship between asymmetric slopes and vertical shear by comparing the downshear and upshear for all of the cases analyzed. The shear vectors were obtained from the Statistical Hurricane Intensity Prediction Scheme (SHIPS; DeMaria and Kaplan 1994) archive. Since the azimuthal resolution of our slope data was 5°, the directions of the shear vectors were rounded to the nearest 5°. Comparing the downshear and upshear slopes did reveal an expected tendency for the eyewall to tilt downshear. For the 50 cases in which the slope data were available upshear and downshear, the downshear slope was greater in 30 cases (60%) by an average of 10.3° ($p < 0.05$).

Next, we filtered out low-shear cases in order to see whether the mean difference in upshear versus downshear slope changed in cases where the shear was significant. This was accomplished by separating the data into cases where the SHIPS shear was greater than 10 kt and cases where the shear was less than 10 kt. The mean difference in slope was indeed higher in the high-shear cases (15.0°) than in the low-shear dataset (3.7°), and a higher percentage of cases tilted downshear in the high-shear set (66%) than in the low-shear set (52%). However, the difference in slope was not significant at the 95% level, contributed to by the small sample size of each subset. Because this relationship may be stronger on a case-by-case basis, the example of Hurricane Earl (2010) in section 4 highlights the effects of shear on eyewall slope. Although not included in this study, future work may be able to quantify the slope asymmetries induced by processes other than shear, such as beta-gyre-induced tilt (Fiorino and Elsberry 1989; Wang and Holland 1996), particularly in the low-shear cases.

f. Slope differences between the upper and lower troposphere

Another slope asymmetry that was observed in some cases was the difference in slope between the lower and upper troposphere. Prior studies have suggested that this vertical difference in slopes is due (for momentum surfaces) to the vertical differences in core temperature and pressure gradients, and the related weakening of cyclonic vorticity with height (Hawkins and Imbembo 1976; Corbosiero et al. 2005, Powell et al. 2009). Stern and Nolan (2012), however, discussed how the strength and height of the hurricane’s warm core do not necessarily directly determine the slope of the RMW. SN09 discussed this asymmetry (for the slope of the RMW) within the context of MPI theory, showing that the slope of a momentum surface varies as radius cubed. We next examined the difference in slopes between the lower and upper troposphere, as well as the slope–intensity relationship between the lower and upper parts of the storm. To accomplish this, the eyewall (from 2 to 11 km) was separated into two equally spaced vertical layers: a lower layer from 2 to 6.5 km and an upper layer from 6.5 to 11 km. Because there are fewer data points for each section than for the whole eyewall (especially in the upper eyewall where the 20-dBZ surface did not always extend to 11 km), it was determined that least squares regression was not the best methodology for calculating the slopes for this comparison. Instead, these slopes were assessed by calculating $\frac{\partial z}{\partial r}$ for each segment from 2 to 11 km (2–6.5 km for lower and 6.5–11 km for upper) to the top of the 20-dBZ surface, and
then averaging the segments over the layer of interest. While this method was found to not significantly alter the composite results for the slope of the eyewall, it did produce a slightly lower overall mean slope than the regression technique (52° as opposed to 58°). Such differences can be expected because of the fact that this is a linear average, whereas linear regression minimizes the squared error. Thus, the actual numerical value of this slope must be interpreted carefully when compared to the overall slope in previous sections, but can be used for comparing the upper and lower parts of the eyewall.

After calculating the slopes for the lower and upper parts of the storm and once again averaging around the BT times, we indeed found a tendency for the upper part of the eyewall to slope outward more than the lower part. Of the 53 BT points examined, 39 (74%) had a larger slope in the upper part of the eyewall. In addition, this methodology produced an average slope of 46° (56°) in the lower (upper) layer. It is possible that this difference ($p < 0.05$) is partially responsible for the differences between the slope distribution in this study and the distribution of RMW slope in SN09, because that study only considered data up to 8 km. However, as discussed previously, the edge of the eyewall updraft as defined by reflectivity is likely a different physical measure of core structure than the RMW.

Interestingly, there was also a significant difference in the slope–intensity relationship between the lower and upper layers. Figure 9 shows scatterplots of intensity versus the lower and upper slopes. The lower slope was correlated with intensity at approximately the same level as the overall slope ($r = -0.39; p < 0.01$, with wind). However, the upper slope showed a statistically insignificant correlation with intensity ($r = 0.12; p = 0.37$ with wind). This result suggests that it is the lower slope that is primarily responsible for the overall correlation between slope and intensity observed in the current study.

### 4. Case studies

We next examine several case studies to look at details of how the eyewall slope evolves within the life cycle of three storms: Felix (2007), Ivan (2004), and Earl (2010). These cases highlight the complexity of the slope–intensity relationship and also provide insight into some of the key physical processes that may be responsible for changes in the slope, such as changes in ocean heat content (Felix), eyewall replacement cycles (Felix and Ivan), and vertical shear (Earl). This analysis compares the observations from the composite analyses (particularly regarding the effects of shear and eyewall replacement cycles) to changes in the structure of individual storms.


Felix was a category 5 hurricane that moved through the Caribbean Sea in early September 2007 and was also one of the most rapidly intensifying hurricanes on record. According to the National Hurricane Center’s tropical cyclone report (http://www.nhc.noaa.gov/2007atlant.shtml), Felix intensified by 115 kt in the 48-h period from 0000 UTC 1 to 0000 UTC 3 September, including an 85-kt 24-h intensification period from 2 to 3 September. This rate of intensification was second only to Wilma (2005) for Atlantic hurricanes. Felix was one of the cases analyzed where there was a relatively high correlation between slope and intensity ($r = 0.93; p = 0.07$).

Figure 10 shows a series of azimuthally averaged radar reflectivity images for the 24-h period during which most of the intensification took place, from 2125 UTC 1 September to 2254 UTC 2 September. Figure 10 also illustrates how the eyewall became more upright and extended higher into the troposphere as the inner-core developed and the storm strengthened. For example, at 2125 UTC 1 September, the slope was 73° and the convection in the eyewall (as defined by the 20-dBZ contour) only extended up to about 6 km (Fig. 10a). By
2254 UTC 2 September, the slope had decreased to 39°, and the convection in the eyewall extended well above 10 km (Fig. 10d). During this time, the diameter of the eye (at $z = 2$ km) decreased from approximately 29 to 15 km. Figure 11 also illustrates how Felix’s eyewall became more upright as the inner core developed and the storm strengthened. It can be seen in Fig. 11 that the eyewall slope decreased significantly as the wind and temperature anomaly increased (as measured by aircraft reconnaissance) in the strengthening part of Felix’s life cycle.

While there were likely several factors that led to the decrease in eyewall slope and sudden intensification of Felix, the favorable environment as a result of low shear (as analyzed by SHIPS) and high ocean heat content (OHC) likely played a role. It has been shown that areas of high OHC can play a major role in the intensification of TCs (e.g., Shay et al. 2000; Hong et al. 2000). OHC is currently included in the SHIPS model and has been demonstrated to be a useful predictor of TC intensity (DeMaria et al. 2005). Mainelli et al. (2008) found that accounting for high OHC values ($\geq 60$ kJ cm$^{-2}$) is especially useful for improving intensity forecasting. The OHC during Felix (Shay and Brewster 2010; Meyers 2011) is shown in Fig. 12, and white circles show the path of the storm and illustrate the changes in slope (circle size) as the storm moved (Fig. 12a is all points; Fig. 12b shows only BT points). The slope was large (less upright) early in the period as the storm was weaker and moving over a region of relatively low OHC ($< 50$ kJ cm$^{-2}$). As Felix moved past a sharp gradient of OHC into a region with much higher OHC (75–100 kJ cm$^{-2}$) the slope became more upright quickly, corresponding with the sharp increase in intensity. Thus, it appears that as the storm began to strengthen in the favorable environment, the increase in OHC may have supported an increase in buoyancy, more intense convection, and stronger updrafts in the core. The stronger updrafts were further evidenced by Felix’s tropical cyclone report, which stated that the NOAA airplane “encountered extreme turbulence and vertical motions” during an eye penetration shortly after 0000 UTC 3 September (Beven 2012). With relatively light vertical shear ($< 10$ kt, as analyzed by the SHIPS model)
throughout the intensification period, the updrafts were able to strengthen and grow without being tilted significantly downshear, and the slope became very upright as the storm intensified.

Interestingly, at the end of the observation period for Felix, the eyewall became more sloped, although this observation was not resolved by the BT averaging (Figs. 10a,b). The increase in slope corresponded with the slight weakening in BT intensity around this time. Microwave imagery from the Naval Research Laboratory’s (NRL) Monterey Marine Meteorology Division archive (http://www.nrlmry.navy.mil/TC.html) and radar observations (Fig. 13) revealed a possible secondary eyewall developing around this time, which may have diverted inflow to the secondary eyewall and weakened the original eyewall updraft, leading to the greater slope.


Ivan (2004) was a long-lived Cape Verde hurricane that earned the distinction of being the lowest-latitude Atlantic major hurricane in recorded history (Stewart 2012). Ivan was the most observed storm in our dataset, with 24 different flight legs corresponding to nine different BT times from 1800 UTC 7 September to 0000 UTC 16 September. Figure 14 shows microwave images of Ivan from 1435 UTC 11 September until 1317 UTC 13 September. Throughout its life span, Ivan experienced several periods of intensification and weakening, with one period of weakening associated with an eyewall replacement cycle on 10 September. Ivan then restrengthened and plateaued from 11 through 13 September, reaching a peak of 910 hPa early on 12 September. As Ivan moved into the Gulf of Mexico on 13 September, a combination of inner-core processes [such as another eyewall replacement cycle (Fig. 14) discussed later] and unfavorable environmental conditions [such as dry continental air, as discussed in Stewart (2012)] started another weakening trend. It appears that these multiple fluctuations in inner-core structure and intensity likely affected the slope–intensity relationship, as the analysis...
Fig. 13. (a) Special Sensor Microwave Imager (SSM/I) 85-GHz microwave images of Hurricane Felix at 1238 UTC 3 Sep 2007. [Figure provided by the NRL Monterey Marine Meteorology Division (http://www.nrlmry.navy.mil/TC.html).] (b) NOAA P3 radar reflectivity at 5 km for Felix at 1227 UTC 3 Sep 2007. Note how both images seem to show the development of a secondary eyewall (highlighted by arrows).
here showed that Ivan was one of the cases with no apparent relationship between slope and intensity \((r = 0.19; p = 0.63)\). However, Fig. 15 illustrates how Ivan’s eyewall slope changed with the maximum wind speed and eye temperature anomaly. The eyewall became slightly more upright during the initial strengthening period and became slightly less upright prior to the slight weakening period.

One feature that stands out from the time series of slope measurements in Ivan is the sharp increase in slope \((-15^\circ)\) at the end of the time period shown, on 13 September. At the same time, the diameter of the eye increased from 6 to approximately 40 km. This increase occurred just prior to the weakening over the Gulf of Mexico. Microwave imagery (Fig. 14) and azimuthally averaged radar images of Ivan on 12 September.
(Figs. 16a,b) show a secondary eyewall forming in the core of the storm and the inner eyewall weakening. By the afternoon of 13 September, the inner eyewall had collapsed, leading to a widening of the eye and a significant increase in the slope. This can be seen in the radar images for 13 September (Fig. 16c) as well as the microwave imagery (Fig. 14). As discussed earlier and in SN09, such eyewall replacement cycles tend to disrupt the overall size–intensity relationship. Assuming the same principle applies to the slope–intensity relationship, this likely explains the low correlation for Ivan. Despite the complete collapse of the inner eyewall at the time of the slope measurement on the afternoon of 13 September, $P_{\text{min}}$ had not yet risen, and was a very low 913 hPa. However, 9 h later $P_{\text{min}}$ had risen to 924 hPa, and continued to rise on 14 September. This slope measurement on 13 September ended up being a major outlier in the intensity–slope relationship (both for Ivan and overall in the cases analyzed), because the eyewall slope changed quickly, seemingly before $P_{\text{min}}$ had time to respond. This again suggests that there is a lag between slope and intensity in some cases (as discussed earlier). This aspect of slope changes in Ivan also shows how sampling can significantly affect the statistical relationship. If the slope measurements had been made 6–12 h later, the corresponding observed $P_{\text{min}}$ would likely have been higher, closer to the expected pressure–slope relationship (Fig. 5). However, the measurement was uniquely positioned in the time window where the core had collapsed but $P_{\text{min}}$ had not yet risen. While providing excellent insight into the relative phase of these structural changes, this sampling time may have a significant impact on the statistical relationships in this case.

c. Hurricane Earl (2010)

Earl is a unique case for study because of the large amount of data collected from 29 August to 4 September during GRIP. Earl was another long-tracked Cape Verde storm that affected many of the islands in the northern Caribbean and southwest Atlantic before brushing the U.S. east coast. It reached its peak intensity of 927 hPa and 125 kt early on 2 September. Observations show that the eyewall became more upright as the storm strengthened over the warm waters near the Caribbean Islands and, then, became less upright as the storm moved into the colder waters of the north-central Atlantic. Factors other than lower OHC, such as increased southwesterly shear, also contributed to the weakening of Earl late in the time period (Cangialosi 2012). Earl was another case with a marginally statistically significant relationship between eyewall slope and intensity ($r = -0.72; p = 0.10$), although as mentioned earlier this relationship was not included in the composite results since it was not an azimuthal-average slope.

One of the interesting features observed in Earl was the apparent impact of the synoptic environment on the core structure and eyewall slope, particularly through the effects of vertical shear. Figure 17a shows a radar cross section of Earl at 1826 UTC 30 August. In Fig. 17a, we see that the right (south) edge of the eyewall slopes more than the left (north) edge of the eyewall, and the left (north) edge in fact slopes slightly inward with height in this case. SHIPS data show that there was north-northwesterly vertical shear (11.7 kt) over Earl at 0000 UTC 30 August. This shear was likely responsible for the slight tilting over of the eyewall on the northern side, as well as the higher slope on the southern side. This difference in slope is seen again in another north–south cross section slightly later at 1948 UTC (Fig. 17b). These observations correspond well with the tendency for greater downshear slope discussed in the composite results. The more upright northern eyewall also corresponded with a convective burst in the northern and eastern eyewall, as seen in microwave imagery (Fig. 18).
It is possible that this asymmetry was partially caused by interaction with the upper-level trough to the north and northwest of the storm, as illustrated by the National Centers for Environmental Prediction (NCEP) Climate Forecast System Reanalysis (CFSR) data (Saha et al. 2010) in Fig. 19. Such trough interactions have been shown to be favorable for intensification in some cases (e.g., Colón and Nightingale 1963), possibly resulting from the rising motion induced by the momentum flux (Molinari and Vollaro 1989). For Earl, this enhanced rising motion (as possibly manifested as stronger convection) may have strengthened the updraft and caused it to become more upright. It should be noted that other factors could also have led to the convective burst, such as a local SST maximum and internal vortex dynamics, but the trough interaction is a reasonable possible mechanism in light of the shear effects observed.

Another interesting note about Earl is the fact that the storm underwent an eyewall replacement cycle on 31 August, but slope measurements do not exist around this time. Thus, the measurements for this storm avoided some of the disruption of the intensity–slope relationship due to secondary eyewall formation, such as seen in Ivan. Once again, this illustrates how important the timing of the observations is in assessing relationships between various changes in the structure of TCs.

5. Concluding summary

The results of this study indicate that the slope of the edge of the eyewall is an important aspect of the TC inner-core structure. While this quantitative observational analysis of slope is limited by data availability and quality, it does allow for explorations of physical relationships within the TC core. While there is a wide spread in the eyewall slopes, a slope of $65^\circ - 70^\circ$ is most common. Looking at the relationship between eyewall slope and intensity, we find that there is a statistically significant relationship. This relationship suggests that the slope of the edge of the eyewall may be fundamentally different in some ways from other measurements of eyewall slope, such as the slope of the RMW (SN09). However, some of the relationships that apply to the slope of RMW also are seen in the slope of the eyewall, such as that between size and slope and the effects of eyewall replacement cycles. The relationship between
slope and intensity applies almost equally within 12 h on either side of the time of the slope measurement. Analyses of TC slope asymmetries show that the downshear slope tends to be greater than the upshear slope, and the slope also tends to be greater in the upper troposphere.

Over an individual storm's life span, the relationship between slope and intensity is less clear, but more interesting from a physical perspective. Some storms have a strong relationship between slope and intensification, while others show little or no relationship. The three case studies highlight possible physical reasons for this variability. The edge of the eyewall becomes more upright as the storm intensifies, due to favorable environments for stronger convection (such as the low shear and high OHC seen in the Felix case) or the response to forcing from a trough (Earl). However, inner-core structural changes may lead to a lag between changes and slope and changes in intensity. This was clearly shown in Ivan, where an eyewall replacement cycle caused the upright inner eyewall to collapse, but the intensity did not respond for several hours. Earl also showed how the inner core sometimes responds to environmental shear to produce asymmetries such as the inward-sloped edge of the eyewall induced by the northerly shear. This case study also highlighted the systematic tendency for

Fig. 17. Radar cross section of Hurricane Earl at (a) 1826 and (b) 1948 UTC 30 Aug 2010. These cross sections are approximately north-south. Note how the left (north) edge of the eyewall slopes inward (north-south) with height, likely caused by northerly vertical shear.

Fig. 18. SSM/I 85-GHz microwave imagery of Hurricane Earl at 2107 UTC 30 Aug 2010. Note the strong convection in the north and east eyewall. The black line highlights the approximate location of the radar cross section from APR-2 shown in Fig. 17. [Figure provided by the NRL Monterey Marine Meteorology Division (http://www.nrlmry.navy.mil/TC.html).]

Fig. 19. CFSR 300-hPa height anomaly from zonal mean (over this basin) and mean sea level pressure, showing Hurricane Earl interacting with the trough to its north.
downshear tilt. Eyewall slope demands further examination to determine if the relationships found here through composites, as well as case studies, can be utilized to better understand short-term intensity or structure change once the sample size has reached critical mass.

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